§9.3: Paired Data Tests and "Blocking"

"Paired Data" Tests are for joint r.v. X & Y that are not independent.

↳ These ideas will be discussed further in the next chapter on "Analysis of Variance" (ANOVA)

Testing difference between dependent population means.

Example: Experiment measuring "before" vs. "after" values.

Experiments making two related measurements on each subject.

Def: Paired Data comes from sampling (X, Y) as a pair

$$(X_1, Y_1) \quad (X_2, Y_2) \quad \cdots \quad (X_n, Y_n)$$

Instead of considering $\mu_X - \mu_Y$ we will look at μ_D where $D_i = X_i - Y_i$ is difference in each pair.

(Note: $\mu_D = E[D] = E[X-Y] = E[X] - E[Y] = \mu_X - \mu_Y$)

"Paired t-Test"

Setup: The pair (X, Y) is sampled n times.

Compute differences $D_i = X_i - Y_i$ in each pair and average to get "sample pair mean diff." $\bar{d}$

↳ Perform regular t-test on $\bar{d}$:

Hypothesis Test:

$$H_0: \mu_D = \Delta \quad \xleftrightarrow{\text{equivalent to}} \quad H_0: \mu_X - \mu_Y = \Delta$$

Test Statistic:

$$\frac{\bar{D} - \Delta}{S_D/\sqrt{n}} \sim t(n-1)$$

p-value:

$$pt\left(-\frac{|\bar{d} - \Delta|}{s_d/\sqrt{n}},\ n-1\right)$$

(where s_d^2 is sample variance of D)

Benefit of paired testing: If X & Y are not independent then s_D^2 (sample variance of difference) is probably much smaller than $s_X^2 + s_Y^2$

Problem with paired testing: The number of degrees of freedom also gets smaller.

↳ Pooled variance t-Test of $X - Y$ would have $(n+n-2) = 2n-2$ TWICE the number of degrees of freedom!

[When sample data is paired (X_i, Y_i) then usually the decrease in sample variance s_d^2 results in a lower p-value (better p-value) than you would get if you ignore the pairing & do a "two-sample" test with higher degrees of freedom.]

Note: You can also do paired data z-Tests, but usually paired data situations are low # samples... so most often in practice people use paired data t-Tests.

"Blocking"

Paired data tests have such wonderfully low variance that people will sometimes artificially convert "two sample" problems into "paired data".

Def: The "blocking" method is to divide two-sample data X & Y into "blocks"

$$B^{(1)} = \left(\{X_1^{(1)}, X_2^{(1)}, \dots, X_{k_1}^{(1)}\}, \{Y_1^{(1)}, Y_2^{(1)}, \dots Y_{j_1}^{(1)}\} \right)$$

$$\vdots$$

$$B^{(n)} = \left(\{X_1^{(n)}, \dots, X_{k_n}^{(n)}\}, \{Y_1^{(n)}, \dots Y_{j_n}^{(n)}\} \right)$$

and then perform paired data tests on means

$$(\bar{x}^{(1)}, \bar{y}^{(1)}), (\bar{x}^{(2)}, \bar{y}^{(2)}), \dots, (\bar{x}^{(n)}, \bar{y}^{(n)})$$

The lower variance of paired data often yields a lower (better) p-value.

But you should only do this if you have good justification for dividing the data into your chosen blocks.